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Nuttier bubbles

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Abstract: We construct new explicit solutions of general relativity from double analytic continuations of Taub-NUT spacetimes. This generalizes previous studies of 4-dimensional nutty bubbles. One 5-dimensional locally asymptotically AdS solution in particular has a special conformal boundary structure of $AdS_3 \times S^1$. We compute its boundary stress tensor and relate it to the properties of the dual field theory. Interestingly enough, we also find consistent 6-dimensional bubble solutions that have only one timelike direction. The existence of such spacetimes with non-trivial topology is closely related to the existence of the Taub-NUT(-AdS) solutions with more than one NUT charge. Finally, we begin an investigation of generating new solutions from Taub-NUT spacetimes and nuttier bubbles. Using the so-called Hopf duality, we provide new explicit time-dependent backgrounds in six dimensions.

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1. Introduction

Many important problems in physics, such as cosmological evolution or black hole evaporation, involve time in an essential way. Therefore, a key problem in string theory is understanding its behavior in time-dependent backgrounds. In order to carry out this investigation one needs to construct simple enough time dependent-solutions that would provide consistent test-beds on which one could try to address these problems.

'Bubbles of nothing' are smooth, time-dependent[1] vacuum solutions of Einstein's equations and so are consistent backgrounds for string theory, at least at leading order. The characteristic feature of such a solution is that it has a (minimal) area with *no* space inside. For example, a bubble solution can be obtained from a four-dimensional static, spherically symmetric black hole by a double analytic continuation in the time coordinate and some other combination of coordinates on the S^2-section. This way, the sphere is effectively changed into a two-dimensional de Sitter spacetime.

The first example was provided by Witten [1] as the endstate of the decay of the Kaluza-Klein (KK) vacuum. Balasubramanian et al. [11] found a similar process in Anti-de Sitter (AdS) spacetime. That is, a certain orbifold in AdS (analogue of the flat space

[1] There are also time-independent bubbles. Properties of bubbles of nothing in different situations are studied in [1 – 10].

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KK vacuum) decays via a bubble of nothing. This opens the possibility that highly non-perturbative processes in gravity might be described (via AdS/CFT correspondence [12]) as barrier penetration in a dual field theory effective potential.

Recently there has been renewed interest in bubbles of nothing since it was pointed out that they provide new endpoints for Hawking evaporation [13]. Closed string tachyon condensation is at the basis of a topology changing transition from black strings to bubbles of nothing.

In this paper we will extend the work of [5] in higher dimensions, in which so-called 'nutty bubbles' — time-dependent backgrounds obtained by double analytic continuations of the coordinates/parameters of (locally asymptotically flat AdS) NUT-charged solutions — were obtained.[2] The gravitational instantons associated with NUT-charged spacetimes come in two classes: 'nuts' and 'bolts'. In four dimensions the topology of nut is R^4 and the apparent singularity at the origin is nothing but a coordinate singularity of the polar coordinate system — in this context (AdS) Taub-NUT is an extremal background. On the other hand, the (AdS) Taub-Bolt geometry should be thought of as a Euclidean black hole with a NUT charge and non zero temperature. In certain situations the NUT charge induces an ergoregion into the bubble spacetime and in other situations it quantitatively modifies the evolution of the bubble, as when rotation is present.

It was conjectured in [5] that one cannot construct consistent nutty bubble solutions (with only one timelike direction) in higher dimensions because (at that time) no 5 (or higher)-dimensional NUT-charged solutions with more than one NUT charge were known. Recently such generalized NUT-charged solutions were obtained [16 – 18] — from these we are able to provide interesting time-dependent bubble solutions in higher-dimensions.

Our paper is organized as follows: in the next section we construct bubble solutions starting from the 5-dimensional solutions presented in [16, 18]. In five dimensions the Taub-NUT solutions have only one nut parameter. Moreover there is a restriction that connects the value of the nut parameter to the cosmological constant. The bubbles are obtained by performing appropriate double analytic continuations of the coordinates. While our focus is primarily on the asymptotically AdS solutions, we also provide non-trivial bubble solutions that are asymptotically dS as well as a 5-dimensional non-asymptotically flat solution. Remarkably, we find a locally asymptotically AdS solution with a boundary geometry of $AdS_3 \times S^1$. In the Discussion section we calculate its boundary stress tensor and show that it has two pieces: one that depends on the parameters of the bubble, and the other one which is universal and is reproduced by the universal anomaly contribution to the stress tensor of Yang-Mills theory on $AdS_3 \times S^1$.

In the third section we construct interesting higher dimensional nutty bubbles from some of the 6-dimensional Taub-NUT solutions presented in [16, 17], however we focus on describing in detail only a couple of representative 6-dimensional spaces. In contrast to the lower-dimensional spaces, in higher than 6-dimensions there can be at least two independent nut parameters and, quite generically, there exits a set of constraints that

[2]Intuitively, the NUT charge corresponds to a 'magnetic' type of mass. The Taub-NUT spacetimes and their boundary geometries are relevant for gauge/gravity dualities [14] — see, also, [15] and references therein for other applications.

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relate the values of these nut parameters to the cosmological constant. We can analytically continue the nut parameters independently, as long as we can still satisfy the constraints (or their analytically continued avatars). We consider first the case when the base space of the circle fibration characteristic of the Taub-NUT solution is $S^2 \times S^2$. In this case there are two independent nut parameters only if the cosmological constant vanishes. We also treat in appendix A the case in which the base space is of the form $M_1 \times M_2$, where the 2-dimensional factors M_i are distinct (in which case the cosmological constant is non-vanishing). In six dimensions there exists a different class of cosmological Taub-NUT solutions, which are characterized by one nut parameter only. The fibration is constructed over the first 2-dimensional factor M_1 and we consider the warped product with M_2. The novelty of this type of solution is that the warp factor depends non-trivially on the cosmological constant and the NUT charge. The time-dependent bubble solutions are obtained by double analytic continuations of the coordinates *and* the nut parameter. Finally, we also present a method to generate new time-dependent solutions by using Hopf-dualities. Essentially, Hopf duality is a T-duality applied along the U(1)-fibre characteristic to the Taub-NUT-like fibrations [19 − 22]. We apply this method to some of our 6-dimensional bubble solutions to generate new time-dependent backgrounds. We end our paper with a discussion section. In appendix A, we present other examples of 6-dimensional time-dependent solutions, constructed starting from the Taub-NUT spaces whose basis are of the form $S^2 \times T^2$, respectively $T^2 \times T^2$. For convenience, in appendix B we present a derivation of the Hopf-duality rules.

2. Five dimensional ‘nutty’ bubbles

In four dimensions the usual Taub-NUT construction corresponds to a circle-fibration over a base space that is a two-dimensional Einstein-Kȧhler manifold. This base space is usually taken to be the sphere S^2 — however, it can also be the torus T^2 or the hyperboloid H^2. In five dimensions the corresponding base space is three dimensional and consequently the above construction is not straightforward. A five-dimensional Taub-NUT space built as a ‘partial’ fibration over a two-dimensional Einstein-Kähler space was given in [16, 18]. However, in five dimensions there is a constraint on the possible values of the nut charge and the cosmological constant. The effect of this constraint is such that for a circle fibration over the sphere, S^2, the cosmological constant can take only positive values, for a fibration over the torus, T^2, the cosmological constant must vanish, while in the case of a fibration over the hyperboloid, H^2, the cosmological constant can have only negative values.[3] It is worth mentioning that one cannot simultaneously set the nut charge and/or the cosmological constant to zero — i.e. there is no smooth limit in which one can obtain five dimensional Minkowski space in this way. However, in the Discussion section, we will present ways to evade this situation.

2.1 Nutty bubbles in AdS

As noted above, the cosmological constant can be negative ($\Lambda = -\frac{6}{l^2}$) only in the case of

[3]We are considering here the Lorentzian sections of the metric.

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a fibration over the hyperboloid H^2. The metric is

$$ds^2 = -F(r)(dt - 2n\cosh\theta d\phi)^2 + F^{-1}(r)dr^2 + (r^2+n^2)(d\theta^2 + \sinh^2\theta d\phi^2) + r^2dy^2, \quad (2.1)$$

where

$$F(r) = \frac{4r^4 + 2l^2r^2 - 16ml^2}{l^2(4r^2+l^2)}. \quad (2.2)$$

Moreover, there is a constraint on the nut parameter $n^2 = \frac{l^2}{4}$, which we already used to simplify the expression of $F(r)$. If we analytically continue the coordinate $t \to i\chi$ and then perform further analytic continuations in the H^2 sector, the following distinct metrics are obtained:

$$\begin{aligned} ds^2 &= F(r)(d\chi + l\cos t d\phi)^2 + F^{-1}(r)dr^2 + (r^2 + l^2/4)(-dt^2 + \sin^2 t d\phi^2) + r^2dy^2 \\ ds^2 &= F(r)(d\chi + l\sinh\theta dt)^2 + F^{-1}(r)dr^2 + (r^2 + l^2/4)(d\theta^2 - \cosh^2\theta dt^2) + r^2dy^2, \\ ds^2 &= F(r)(d\chi + l\cosh\theta dt)^2 + F^{-1}(r)dr^2 + (r^2 + l^2/4)(d\theta^2 - \sinh^2\theta dt^2) + r^2dy^2, \\ ds^2 &= F(r)(d\chi + le^{\theta}dt)^2 + F^{-1}(r)dr^2 + (r^2 + l^2/4)(d\theta^2 - e^{2\theta}dt^2) + r^2dy^2. \end{aligned} \quad (2.3)$$

They are solutions of the vacuum Einstein field equations with negative cosmological constant $\Lambda = -6/l^2$.

For the last three geometries the coordinate θ is no longer periodic and can take any real value. The geometry in the second bracket is described by a two-dimensional AdS space. As is well known, this space can have non-trivial identifications and so the 2-dimensional sector can describe a 2-dimensional black hole (as in the second and third metrics above), while the first metric describes pure AdS in standard coordinates. Notice however that in this case, the geometry of a fixed (χ, r, y)-slice is AdS_2 modified by the term $F(r)l^2\sinh^2\theta dt^2$ as an effect of the non-trivial fibration over the (θ, t)-sector. This extra term will vanish only at points where $F(r) = 0$ (thence, on the bubble).

After a coordinate transformation the last three metrics can be written in a compact form

$$\begin{aligned} ds^2 &= F(r)(d\chi + lxdt)^2 + F^{-1}(r)dr^2 + (r^2 + l^2/4)\left(\frac{dx^2}{x^2+k} - (x^2+k)dt^2\right) \\ &\quad + r^2dy^2, \end{aligned} \quad (2.4)$$

with $k = -1, 1, 0$. They are all locally equivalent under changes of coordinates. However, depending on the identifications made, the global structure can be quite different.

The quartic function in the numerator of $F(r)$ can have only two real roots — for $m > 0$, one is positive (denoted by r_+) and the other one is negative (denoted by r_-):

$$r_\pm = \pm\frac{l}{2}\sqrt{\sqrt{1 + 64m/l^2} - 1}. \quad (2.5)$$

The conical singularities at either root of $F(r)$ in the (χ, r)-sector can be eliminated if the periodicity of the χ-coordinate is

$$\beta = \frac{4\pi}{|F'(r_\pm)|} = \frac{2\pi l}{\sqrt{\sqrt{1 + 64m/l^2} - 1}}. \quad (2.6)$$

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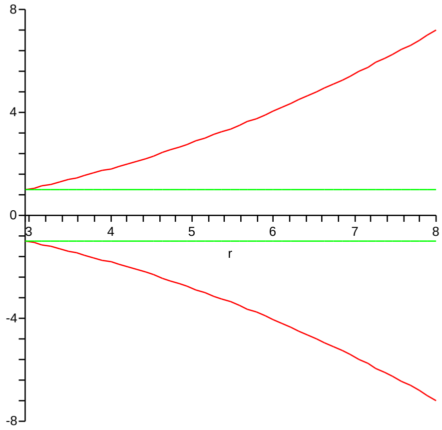

Figure 1: Ergoregion of the topological metric with $k = -1$ in the (r, x) plane for $m = 20$ and $l = 1$. The ergoregion is confined to the infinite regions bounded by the red curves and outside the black hole horizons $|x| = 1$.

Now, for $r > r_+$ (or $r < r_-$) the first three metrics will describe stationary backgrounds. Note that the metric (2.4) is stationary and it possesses the Killing vector $\xi = \frac{\partial}{\partial t}$. The norm of this Killing vector is

$$\xi \cdot \xi = l^2 x^2 F(r) - (r^2 + l^2/4)(x^2 + k),$$

and we find that in general there is an ergoregion iff

$$\left(\frac{4l^2 F(r)}{4r^2 + l^2} - 1 \right) x^2 > k.$$

However, since the expression in the bracket is always negative we find that there exists an ergoregion only if $k = -1$ in which case the following constraint is obtained:

$$|x| < \frac{4r^2 + l^2}{l\sqrt{64m + l^2}}.$$

The ergoregion corresponds to a strip in the (r, x) plane bounded by the horizons located at $|x| = 1$ and two curves that asymptote to 1 for $r \to r_\pm$, while for large values of r the strip will largely broaden (see figure 1) and the curves asymptote to $\frac{4x^2}{l\sqrt{64m+l^2}}$. Also, as it is apparent from the above formula, the strip broadens as m decreases.

In the remaining cases, for $k = 0, 1$ there is no ergoregion.

The asymptotic structure of the above metrics is

$$ds^2 = r^2/l^2(d\chi + lxdt)^2 + l^2/r^2 dr^2 + r^2 \left(\frac{dx^2}{x^2 + k} - (x^2 + k)dt^2 \right) + r^2 dy^2. \qquad (2.7)$$

Now, it is easy to read the boundary geometry — up to a conformal rescaling factor r^2/l^2, the boundary metric is

$$ds^2 = l^2(d\tilde{\chi} + xdt)^2 + l^2 \left(\frac{dx^2}{x^2 + k} - (x^2 + k)dt^2 \right) + l^2 dy^2. \qquad (2.8)$$

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Here, we use a rescaled coordinate $\tilde{\chi} = \chi/l$. From (2.6) it is easy to see that if $m = \frac{l^2 s^2(s^2+8)}{1024}$ then $\tilde{\chi}$ has periodicity $4\pi/s$, with s an integer. Remarkably, for $s = 1$ the boundary geometry is conformally flat. This can be easily seen from the fact that the boundary metric is the product of a 3-dimensional space of constant curvature (i.e. pure AdS_3) with a line (or a circle if we also compactify the y coordinate). Furthermore, one can also make nontrivial identifications in the AdS_3 sector which turn it into the BTZ black hole. We will have to say more about these solutions in the Discussion section.

Finally, let us consider the first metric from (2.3). Even if formally it can be transformed into the $k = -1$ metric by a coordinate transformation,[4] if ϕ is periodic then the global structure of these spaces is completely different. The geometry in the (χ, t, ϕ)-sector resembles the usual Hopf-type fibration. The χ-circle is now fibred over the circle described by ϕ. However, the fibration is twisted as a function of time. At $t = 0$ we have a pair of orthogonal circles provided we define χ appropriately. As time increases we have the χ-circle twisting around relative to the ϕ-circle, while the ϕ-circle is getting bigger. The latter reaches a maximum, and then begins to shrink. However the χ-circle is still twisting, and by the time the ϕ-circle has shrunk back to zero, the χ-circle has twisted only 'halfway' round. Over this cycle the integral $\int d(l\cos t d\phi)$ is well-defined, and it equals $4\pi\ell$ since we are integrating t from 0 to π. This will set the periodicity of χ to be $4\pi l/s$, where s is an integer. Recall now that the quartic function $F(r)$ can have only two real roots, one positive (r_+) and one negative (r_-) for $m > 0$. If ϕ is an angular coordinate with period 2π, then in order to eliminate the Misner string singularity we require that the period $\beta = 4\pi/|F'(r_\pm)|$ be equal to $4\pi l/s$, where s is some integer. This further restricts the value of the mass parameter such that $m = \frac{l^2 s^2(s^2+8)}{1024}$.

For $r > r_+$ (or $r < r_-$), this metric describes then a bubble located at $r = r_+$, which expands from zero size to a finite size and then contracts to zero size again. All the spacetime events are causally connected with each other. Near the initial expansion (or the final contraction) the scale factor is linear in time and the spacetime expands or contracts like a Milne universe. The boundary geometry for this bubble spacetime is given by

$$ds^2 = (d\chi + l\cos t d\phi)^2 + l^2(-dt^2 + \sin^2 t d\phi^2) + l^2 dy^2, \tag{2.9}$$

where χ is periodic with period $4\pi l/s$.

2.2 Nutty bubbles in dS

The Taub-NUT ansatz that we shall use in the construction of these spaces is the following:

$$ds^2 = -F(r)(dt - 2n\cos\theta d\varphi)^2 + F^{-1}(r)dr^2 + (r^2+n^2)(d\theta^2 + \sin^2\theta d\varphi^2) + r^2 dz^2. \tag{2.10}$$

[4] Notice however that the range of the x coordinate is different: for the $k = -1$ metric $x \geq 1$, while for the first metric in (2.3) we must take $|x| \leq 1$. Another difference is that the periodicity in χ would be related to a periodicity in ϕ for the first metric, while for the second one, with $k = -1$, it would require us to make t periodic. Therefore we find that while there are no hyperbolic Misner strings for $k = -1$, the fourth metric could have Misner strings.

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The above metric will be a solution of the Einstein field equations with positive cosmological constant $\Lambda = \frac{6}{l^2}$ provided

$$F(r) = \frac{4ml^2 - r^4 - 2n^2r^2}{l^2(r^2+n^2)}, \tag{2.11}$$

where the field equations impose the constraint[5] $4n^2 = l^2$. Notice that, for large values of r, the function $F(r)$ takes negative values and r becomes effectively a timelike coordinate as one should expect in a region outside the cosmological horizon. In order to remove the usual Misner string singularity in the metric, we have to assume that the coordinate t is periodic with period $4\pi l$. If we analytically continue the coordinate $t \to i\chi$ and one of the coordinates in the S^2 sector we obtain the following metrics:

$$\begin{aligned}
ds^2 &= F(r)(d\chi + l\cos\theta dt)^2 + F^{-1}(r)dr^2 + (r^2 + l^2/4)(d\theta^2 - \sin^2\theta dt^2) + r^2dy^2, \\
ds^2 &= F(r)(d\chi + l\cosh t d\phi)^2 + F^{-1}(r)dr^2 + (r^2 + l^2/4)(-dt^2 + \sinh^2 t d\phi^2) + r^2dy^2, \\
ds^2 &= F(r)(d\chi + l\sinh t d\phi)^2 + F^{-1}(r)dr^2 + (r^2 + l^2/4)(-dt^2 + \cosh^2 t d\phi^2) + r^2dy^2, \\
ds^2 &= F(r)(d\chi + le^t d\phi)^2 + F^{-1}(r)dr^2 + (r^2 + l^2/4)(-dt^2 + e^{2t}d\phi^2) + r^2dy^2.
\end{aligned} \tag{2.12}$$

These metrics satisfy the vacuum Einstein field equations with positive cosmological constant $\Lambda = 6/l^2$, where $F(r)$ is given by (2.11). However, for large values of r the function $F(r)$ becomes negative and the signature of the spacetime will change accordingly. To avoid this situation one possibility is to consider two roots of the function $F(r)$ and to restrict the values of the r coordinate such that $F(r)$ is always positive. Namely we restrict the range of the r coordinate such that $r_- < r < r_+$, where $r_\pm$ are two roots of $F(r)$ and in this way we avoid the change in the metric signature. It is easy to see that if $m > 0$ then $F(r)$ has two real roots only if

$$r_\pm = \pm\frac{l}{2}\sqrt{\sqrt{1+64m/l^2} - 1}\,.$$

Fortunately, the conical singularities at the roots of $F(r)$ in the (χ, r)-sector can both be eliminated in the same time if we choose the periodicity of the χ-coordinate to be given by

$$\beta = \frac{4\pi}{|F'(r_\pm)|} = \frac{2\pi l}{\sqrt{\sqrt{1+64m/l^2} - 1}}\,.$$

To eliminate the Misner string singularity in the first metric in (2.12), we require that the period β be equal to $4\pi l/s$, where s is some integer. This further restricts the value of the mass parameter such that $m = \frac{l^2s^2(s^2+8)}{1024}$. Notice that, locally, all these metrics are equivalent, being related by coordinate transformations. However, these spaces will be equivalent globally only if the coordinate ϕ is unwrapped. At every fixed (χ, r, y) the geometry is that of a perturbed two-dimensional de Sitter spacetime as an effect of the non-trivial fibration.

To better understand the geometry in the (χ, r, y) sector let us focus on a section with t, ϕ held fixed. Then the metric in the (χ, r, y) -sector becomes

$$ds^2 = F(r)d\chi^2 + F^{-1}(r)dr^2 + r^2dy^2.$$

[5] In the following we shall use this constraint to eliminate n from the metric.

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We restrict the values of the r-coordinate between the two roots of $F(r)$ and since they have the same magnitude, we shall take $r_+^2 = r_-^2 = r_0^2$. Then, it is easy to see that we can write

$$F(r) = (r_0^2 - r^2)\frac{4r^2 + 2l^2 + 4r_0^2}{l^2(4r^2 + l^2)} = (r_0^2 - r^2)f(r),$$

where $f(r)$ is strictly positive everywhere. Now if we make the following change of coordinates

$$r^2 = r_0^2(1 - x^2),$$

the metric in the (χ, r, y) sector becomes

$$ds^2 = \frac{dx^2}{(1 - x^2)f(r_0\sqrt{1 - x^2})} + r_0^2(1 - x^2)dy^2 + r_0^2 x^2 f(r_0\sqrt{1 - x^2})d\chi^2.$$

A further change of coordinates $x = \sin\psi$ will bring it in the form:

$$ds^2 = \frac{d\psi^2}{f(r_0\cos\psi)} + r_0^2\cos^2\psi dy^2 + r_0^2\sin^2\psi f(r_0\cos\psi)d\chi^2,$$

where

$$f(r_0\cos\psi) = \frac{1}{l^2}\left[1 + \frac{4r_0^2}{4r_0^2\cos^2\psi + l^2}\right].$$

It can be easily seen that the geometry in this sector is one of a deformed 3-sphere. We conclude that our bubble metrics describe non-trivial fibrations of a 3-sphere over a 2-dimensional dS space.

If the coordinate ϕ is periodic, the circle geometry that it describes will evolve differently for each of the above geometries. For instance, for the second metric in (2.12) the evolution is that of a circle that begins with zero radius at $t = 0$ and then expands exponentially as $t \to \infty$, while for the third metric we obtain a de Sitter evolution of a circle with exponentially large radius at $t \to -\infty$ that exponentially shrinks to a minimal value and then expands again. Finally the fourth geometry describes the evolution of a circle which begins with zero radius at $t \to -\infty$ and then expands exponentially as $t \to \infty$. Similar with the four-dimensional situation considered in [5], a null curve in a geometry for which the bubbles are expanding has $|\dot\phi| \leq e^{-t}|\dot t|$ at late times, where the overdot refers to a derivative with respect to proper time. Hence, observers at different values of ϕ will eventually lose causal contact. On the other hand null rays at fixed ϕ and y obey the relation

$$\dot r^2 + V(r) = 0, \tag{2.13}$$

where $V(r) = p_\chi^2 + 4p_y^2 F(r)/r^2 - 4E^2 F(r)/(4r^2 + l^2)$ is an effective potential, $p_\chi = \dot\chi F$ is the conserved momentum along the χ direction, $p_y = r^2\dot y$ is the conserved momentum along the y direction and $E = (r^2 + l^2/4)\dot t$ is the conserved energy. Generically, if $p_y = 0$ then the null geodesics oscillate between some minimal and maximal values of r, which can be chosen to be within the admitted range of r. Hence observers at any two differing values of r can be causally connected. However, if the observers are at different values of y

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then the effective potential diverges at $r = 0$, which means that there will be two regions that can be causally disconnected. It is easy to check that it is possible for observers at different values of χ, respectively y to be causally connected at fixed values of r.

Let us notice that the first metric from (2.12) is stationary since it possesses the Killing vector $\xi = \frac{\partial}{\partial t}$. The norm of this Killing vector is

$$\xi \cdot \xi = l^2 F(r) - (r^2 + l^2/4 - l^2 F(r)) \sin^2 \theta \tag{2.14}$$

and so it becomes spacelike unless $\hat{\theta}(r) \leq |\theta| \leq \pi - \hat{\theta}(r)$. Here, we use the notation

$$\hat{\theta}(r) = \tan^{-1} \left(\frac{2l\sqrt{F(r)}}{\sqrt{4r^2 + l^2}} \right). \tag{2.15}$$

As in the four-dimensional case [5] these limits will describe an 'ergocone' for the space-time. The above angle vanishes at $r = r_\pm$ while it attains a maximum value at $r = 0$.

2.3 Nutty Rindler bubbles in flat backgrounds

We can also obtain NUT spaces with non-trivial topology if we construct the circle fibration over a two-dimensional torus T^2,

$$ds^2 = -F(r)(dt - 2n\theta d\phi)^2 + F^{-1}(r)dr^2 + (r^2 + n^2)(d\theta^2 + d\phi^2) + r^2 dy^2 \tag{2.16}$$

where

$$F(r) = \frac{4ml^2 + r^4 + 2n^2 r^2}{l^2(r^2 + n^2)} \tag{2.17}$$

and the constraint equation takes now the form $\Lambda n^2 = 0$. Consistent Taub-NUT spaces with toroidal topology exist if and only if the cosmological constant vanishes. The Euclidean version of this solution, obtained by analytic continuation of the coordinate $t \to it$ and of the parameter $n \to in$, has a curvature singularity at $r = n$. Note that if we consider $n = 0$ in the above constraint we obtain the AdS/dS black hole solution in five dimensions with toroidal topology.

If the cosmological constant vanishes, then we can have $n \neq 0$ and the metric becomes

$$ds^2 = -F(r)(dt - 2n\theta d\phi)^2 + F^{-1}(r)dr^2 + (r^2 + n^2)(d\theta^2 + d\phi^2) + r^2 dy^2, \tag{2.18}$$

where

$$F(r) = \frac{4m}{r^2 + n^2}. \tag{2.19}$$

The asymptotic structure of the above metric is given by

$$ds^2 = \frac{4m}{r^2}(dt - 2n\theta d\phi)^2 + \frac{r^2}{4m} dr^2 + r^2(d\theta^2 + d\phi^2 + dy^2). \tag{2.20}$$

If y is an angular coordinate then the angular part of the metric parameterizes a three torus. The Euclidean section of the solution described by (2.18) is not asymptotically flat and has a curvature singularity localized at $r = 0$. However, let us notice that for $r \leq n$ the

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signature of the space becomes completely unphysical. Hence, for the Euclidean section, we should restrict the values of the radial coordinate such that $r \geq n$.

Consider now the analytic continuations of the coordinates $t \to i\chi$ and $\theta \to -it$ (respectively $\phi \to -it$) in the case of fibration over a torus. We obtain the spacetimes

$$\begin{aligned} ds^2 &= F(r)(d\chi + 2ntd\phi)^2 + F^{-1}(r)dr^2 + (r^2+n^2)(-dt^2+d\phi^2) + r^2dy^2, \\ ds^2 &= F(r)(d\chi + 2n\theta dt)^2 + F^{-1}(r)dr^2 + (r^2+n^2)(d\theta^2-dt^2) + r^2dy^2 \end{aligned} \tag{2.21}$$

whose metrics are locally equivalent under coordinate transformations. However, if one of the coordinates ϕ or θ is periodic then they represent globally different spaces. Another metric — related to the above by coordinate transformations — is a generalization to five-dimensions of the four dimensional nutty Rindler spacetime. Since $F(r)$ has no roots these spaces are not really bubbles. However they still represent interesting time-dependent backgrounds, with metrics given by

$$\begin{aligned} ds^2 &= F(r)(d\chi + nt^2d\phi)^2 + F^{-1}(r)dr^2 + (r^2+n^2)(-dt^2+t^2d\phi^2) + r^2dy^2, \\ ds^2 &= F(r)(d\chi + n\theta^2 dt)^2 + F^{-1}(r)dr^2 + (r^2+n^2)(d\theta^2-\theta^2dt^2) + r^2dy^2. \end{aligned} \tag{2.22}$$

Let us notice that, by analogy with the four-dimensional case, in the first case, the geometry of a slice (r, θ, y) is that of a twisted torus which has a Milne-type evolution. The second geometry describes a static spacetime (with the Killing vector $\xi = \frac{\partial}{\partial t}$) with an ergoregion described by

$$\theta^2 > \frac{r^2+n^2}{n^2F(r)}.$$

For large values of r, the ergoregion includes almost the entire (θ, r) plane except for a strip bounded by two curves, opposite the r-axis, which asymptote to parabolas. For small values of r the strip narrows and the boundary curves asymptote to $\pm n/\sqrt{2m}$. For the first geometry, as $t^2\dot{\phi}^2 \leq \dot{t}^2$ we have $\phi \sim \ln t$ and observers with different values of ϕ can communicate with each other for arbitrarily large t. In the second geometry we obtain $\dot{\theta}^2 \leq \theta^2\dot{t}^2$, i.e. $\theta \sim e^{|t|}$ and we see that there is no restriction as to the maximum change of coordinate θ for points on the null curve as $t \to \pm\infty$ and observers at points with different θ can communicate with each other.

3. Higher dimensional nutty bubbles

We now consider some of the higher dimensional Taub-NUT spaces constructed recently in [16, 18, 17]. As mentioned previously, in the Taub-NUT ansatz the idea is to construct such spaces as radial extensions of U(1)-fibrations over an even dimensional base B endowed with an Einstein-Kähler metric g_B. In a $(2k+2)$-dimensional Taub-NUT space the base factor over which one constructs the circle fibration can have at most dimension $2k$, in which case the metric is

$$F^{-1}(r)dr^2 + (r^2+N^2)g_B - F(r)(dt + 2N\mathcal{A})^2, \tag{3.1}$$

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where t is the coordinate on the fibre and $\mathcal{F} = d\mathcal{A}$ is the Kähler 2-form. Here N is the NUT charge and $F(r)$ is a function of r. More generally, in the even-dimensional cases we can consider circle fibrations over base spaces that can be factorized in the form $B = M_1 \times \ldots \times M_k$ where M_i are Einstein-Kähler manifolds. In this case we can associate a NUT charge N_i for every such base factor M_i. Also, in the above ansatz we replace $(r^2 + N^2)g_B$ with the sum $\sum_i (r^2 + N_i^2)g_{M_i}$ and $2N\mathcal{A}$ by $\sum_i 2N_i\mathcal{A}_i$. In particular, we can use the sphere S^2, the torus T^2 or the hyperboloid H^2 or in general CP^n as factor spaces.

More generally one can consider more general Taub-NUT-like spaces with factorizations of the base space of the form $B = \prod_i M_i \times Y$, where each factor M_i is endowed with an Einstein-Kähler metric g_{M_i} while Y is a general Einstein space with metric g_Y. In these cases one can consider the U(1)-fibration only over the factored space $M = \prod_i M_i$ of the base B and take then a warped product with the manifold Y. Quite generically, we can associate a nut parameter N_i with every such factor M_i and the general ansatz is then given by

$$F^{-1}(r)dr^2 + \sum_i (r^2 + N_i^2)g_{M_i} + r^2 g_Y - F(r)(dt + \sum_i 2N_i\mathcal{A}_i)^2. \tag{3.2}$$

We now consider particular cases of these ansätze. To be more specific we shall focus on a couple of six-dimensional metrics.

3.1 Bubbles in flat backgrounds

In six dimensions the base space is four-dimensional and we can use products of the form $M_1 \times M_2$ of two-dimensional Einstein-Kähler spaces or we can use CP^2 as a four-dimensional base space over which to construct the circle fibrations. If we use products of two dimensional Einstein-Kähler spaces then we can consider all the cases in which M_i, $i = 1, 2$ can be a sphere S^2, a torus T^2 or a hyperboloid H^2. The circle fibration can be constructed over the whole base space $M_1 \times M_2$, in which case we can have two distinct nut parameters associated with each factor M_i or, in the case of metrics with only one nut parameter, just over one factor space M_1, in which case we also take the warped product with M_2 as in (3.2).

We shall consider first the case in which $M_1 = M_2 = S^2$ and assume that the U(1) fibration is constructed over the whole base space $S^2 \times S^2$. Then the corresponding six-dimensional Taub-NUT solution is given by [16]

$$\begin{aligned} ds^2 = & -F(r)(dt - 2n_1 \cos\theta_1 d\varphi_1 - 2n_2 \cos\theta_2 d\varphi_2)^2 + F^{-1}(r)dr^2 \\ & +(r^2 + n_1^2)(d\theta_1^2 + \sin^2\theta_1 d\varphi_1^2) + (r^2 + n_2^2)(d\theta_2^2 + \sin^2\theta_2 d\varphi_2^2), \end{aligned} \tag{3.3}$$

where

$$\begin{aligned} F(r) = & \frac{3r^6 + (l^2 + 5n_2^2 + 10n_1^2)r^4 + 3(n_2^2 l^2 + 10n_1^2 n_2^2 + n_1^2 l^2 + 5n_1^4)r^2}{3(r^2 + n_1^2)(r^2 + n_2^2)l^2} \\ & - \frac{6ml^2 r + 3n_1^2 n_2^2(l^2 + 5n_1^2)}{3(r^2 + n_1^2)(r^2 + n_2^2)l^2}. \end{aligned} \tag{3.4}$$

Here the above metric is a solution of vacuum Einstein field equations with cosmological constant ($\lambda = -\frac{10}{l^2}$) if and only if $(n_1^2 - n_2^2)\lambda = 0$. Consequently, we see that differing values

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for n_1 and n_2 are possible only if the cosmological constant vanishes. For $n_1 = n_2 = n$ the above solution reduces to the six-dimensional solution found and studied in [23 − 28]. In the case of only one nut charge n there are no consistent analytic continuations of the coordinates that lead to acceptable time-dependent metrics with Lorentzian signature [5]. Basically, the reason for this is that if we perform analytic continuations of the coordinates on one factor space S^2 we also have to send $n \to in$, which will force us to analytically continue the coordinates in the second sphere S^2 yielding spaces with two timelike directions. However, if the nut parameters are independent then we can analytically continue the coordinates in one factor M_i only and analytically continue the nut parameter associated with the second factor M_j. This enables us to construct nutty bubble spacetimes in virtually any dimension. For this reason, in what follows we shall look at the case of two different nut charges, that is we set the cosmological constant to zero. Let us consider the Euclidean section, obtained by the following analytic continuations $t \to i\chi$ and $n_j \to in_j$ where $j = 1, 2$:

$$\begin{aligned} ds^2 &= F_E(r)(d\chi - 2n_1 \cos\theta_1 d\varphi_1 - 2n_2 \cos\theta_2 d\varphi_2)^2 + F_E^{-1}(r)dr^2 \\ &\quad + (r^2 - n_1^2)(d\theta_1^2 + \sin^2\theta_1 d\varphi_1^2) + (r^2 - n_2^2)(d\theta_2^2 + \sin^2\theta_2 d\varphi_2^2), \\ F_E(r) &= \frac{r^4 - 3(n_1^2 + n_2^2)r^2 - 6mr - 3n_1^2 n_2^2}{3(r^2 - n_1^2)(r^2 - n_2^2)}. \end{aligned} \tag{3.5}$$

This metric is a solution of the vacuum Einstein field equations without cosmological constant, for any values of the parameters n_1 and n_2. We set $n_1 > n_2$ without loss of generality. In this case in the Euclidean section the radius r cannot be smaller than n_1 or the signature of the spacetime will change. The Taub-nut solution in this case corresponds to a two-dimensional fixed-point set located at $r = n_1$. There is still a curvature singularity located at $r = n_1$. While superficially it would seem that this could removed by setting the periodicity of the coordinate χ to be $8\pi n_1$ (thereby setting $m = m_p = -\frac{n_1^3 + 3n_1 n_2^2}{3}$), a more careful analysis reveals that this nut solution is actually still singular. This is because the nut parameters $n_{1,2}$ must be rationally related, in which case the periodicity of the χ coordinate is $8\pi n_2/k$, where k is an integer. As $n_2 < n_1$ it is not possible to match this periodicity with $8\pi n_1$ for any integer k.

On the other hand, the bolt solution corresponds to $r \geq r_0 > n_1$ and the periodicity is found to be $\frac{4\pi}{|F_E'(n_1)|} = 4\pi r_0$. It is now possible to match it with $8\pi n_2/k$ with k the integer and we obtain $r_0 = \frac{2n_2}{k}$. The bolt solution is then non-singular as long as $r_0 > n_1$, that is for $k = 1$ and $n_1 < 2n_2$.

We are now ready to perform analytic continuations on the sphere factors in order to generate new time-dependent backgrounds. For instance, we can consider $\theta_1 \to it + \frac{\pi}{2}$, which will force us to take $n_1 \to in_1$. We obtain the following time dependent solution:

$$\begin{aligned} ds^2 &= \tilde{F}_E(r)(d\chi + 2n_1 \sinh t d\phi_1 + 2n_2 \cos\theta_2 d\phi_2)^2 + \tilde{F}_E^{-1}(r)dr^2 \\ &\quad + (r^2 + n_1^2)(-dt^2 + \cosh^2 t d\phi_1^2) + (r^2 - n_2^2)(d\theta_2^2 + \sin^2\theta_2 d\phi_2^2), \\ \tilde{F}_E &= \frac{r^4 + 3(n_1^2 - n_2^2)r^2 - 6rm + 3n_1^2 n_2^2}{3(r^2 + n_1^2)(r^2 - n_2^2)}. \end{aligned} \tag{3.6}$$

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More generally, as with the four-dimensional case, after performing appropriate analytic continuations we end up with the following metrics:

$$\begin{aligned}
ds^2 &= \tilde{F}_E(r)(d\chi + 2n_1 \cosh t d\phi_1 + 2n_2 \cos\theta_2 d\phi_2)^2 + \tilde{F}_E^{-1}(r)dr^2, \\
&\quad + (r^2+n_1^2)(-dt^2 + \sinh^2 t d\phi_1^2) + (r^2 - n_2^2)(d\theta_2^2 + \sin^2\theta_2 d\phi_2^2), \\
ds^2 &= \tilde{F}_E(r)(d\chi + 2n_1 e^t d\phi_1 + 2n_2 \cos\theta_2 d\phi_2)^2 + \tilde{F}_E^{-1}(r)dr^2, \\
&\quad + (r^2+n_1^2)(-dt^2 + e^{2t} d\phi_1^2) + (r^2 - n_2^2)(d\theta_2^2 + \sin^2\theta_2 d\phi_2^2), \\
ds^2 &= \tilde{F}_E(r)(d\chi + 2n_1 \cos\theta dt + 2n_2 \cos\theta_2 d\phi_2)^2 + \tilde{F}_E^{-1}(r)dr^2, \\
&\quad + (r^2+n_1^2)(d\theta_1^2 - \sin^2\theta_1 dt^2) + (r^2 - n_2^2)(d\theta_2^2 + \sin^2\theta_2 d\phi_2^2).
\end{aligned} \tag{3.7}$$

They are also solutions of vacuum Einstein field equations with the same function $\tilde{F}_E$ as in (3.6).

While locally all these spaces are equivalent under coordinate transformations, if we compactify the coordinate ϕ_1(respectively θ_1 for the last metric in (3.7)) the global structure and in particular the evolution of the bubble will be different. The bubble will be located at the biggest root r_0 of $\tilde{F}_E(r)$ such that $r_0 > n_2$ and we also restrict the range of the r coordinate such that $r \geq n_2$. Elimination of the Misner string sets the periodicity of the χ coordinate to be $8\pi n_2/k$, which in turn must be matched with the periodicity $4\pi/|\tilde{F}_E'(r_0)|$, introduced after we eliminate any possible conical singularities in the (χ, r) sector. Again we have two solutions: a nut and a bolt.

The nut solution corresponds to $r_0 = n_2$ and in this case the mass parameter is $n_2(3n_1^2 - n_2^2)/3$. Notice that the mass parameter can have either sign. The coordinate χ has periodicity $8\pi n_2$. It is very interesting to note that the fixed-point set of the isometry generated by $\partial/\partial\chi$ is effectively two dimensional. The induced geometry on the 'bubblenut' is a two-dimensional de Sitter space, whose metrics are one of the following

$$\begin{aligned}
ds_2^2 &= (n_1^2 + n_2^2)(-dt^2 + \cosh^2 t d\phi_1^2), \\
ds_2^2 &= (n_1^2 + n_2^2)(-dt^2 + \sinh^2 t d\phi_1^2), \\
ds_2^2 &= (n_1^2 + n_2^2)(-dt^2 + e^{2t} d\phi_1^2), \\
ds_2^2 &= (n_1^2 + n_2^2)(d\theta_1^2 - \sin^2\theta_1 dt^2).
\end{aligned} \tag{3.8}$$

which differ globally but not locally. If the coordinate ϕ_1 is periodically identified then at any fixed time $r = n_2$ is our 'bubblenut': a circle with minimal circumference that expands or contracts. The first three de Sitter geometries above correspond to three different evolutions of this circle: the first geometry describes the evolution of a circle with exponentially large radius at $t \to -\infty$, which shrinks to a minimal value and expands exponentially again for $t \to \infty$; the second geometry describes the evolution of a circle which begins with zero radius at $t = 0$ and expands exponentially, while the third geometry describes a circle that begins with exponentially small radius at $t \to -\infty$ and then expands exponentially. The last geometry is stationary as in these coordinates the metric has a Killing vector $\xi = \partial/\partial t$. The norm of this Killing vector is

$$\xi \cdot \xi = 4n_1^2 \tilde{F}_E(r) - (r^2 + n_1^2 - 4n_1^2 \tilde{F}_E(r)) \sin^2\theta_1$$

so that it will become spacelike unless $\hat{\theta}_1(r) \leq |\theta_1| \leq \pi - \hat{\theta}_1(r)$; here, we used the notation:

$$\hat{\theta}_1(r) = \tan^{-1}\left(\frac{2n_1\sqrt{\tilde{F}_E(r)}}{\sqrt{r^2+n_1^2}}\right).$$

As in the four-dimensional case [5] these limits will describe an 'ergocone' for the spacetime. The above angle vanishes at $r = r_0$ and at infinity, while it attains a maximum value in between.

The bolt solution corresponds to a four-dimensional fixed-point set of the isometry generated by $\partial/\partial\chi$. By solving the above constraint on the possible periodicities of the χ coordinate we obtain the location of the bolt $r_0 = 2n_2/k$. Requiring that $r \geq r_0 > n_2$ implies $k = 1$, in which case the periodicity of the χ coordinate is $8\pi n_2$ and the mass parameter is $m = n_2(15n_1^2 + 4n_2^2)/12$. Notice that in this case the mass parameter is positive. The induced geometry on the 'bubblebolt' is a two-dimensional de Sitter space times a sphere S^2:

$$\begin{aligned}
ds_2^2 &= (n_1^2+4n_2^2)(-dt^2+\cosh^2 t d\phi_1^2) + 3n_2^2(d\theta_2^2+\sin^2\theta_2 d\phi_2^2),\\
ds_2^2 &= (n_1^2+4n_2^2)(-dt^2+\sinh^2 t d\phi_1^2) + 3n_2^2(d\theta_2^2+\sin^2\theta_2 d\phi_2^2),\\
ds_2^2 &= (n_1^2+4n_2^2)(-dt^2+e^{2t} d\phi_1^2) + 3n_2^2(d\theta_2^2+\sin^2\theta_2 d\phi_2^2),\\
ds_2^2 &= (n_1^2+4n_2^2)(d\theta_1^2-\sin^2\theta_1 dt^2) + 3n_2^2(d\theta_2^2+\sin^2\theta_2 d\phi_2^2).
\end{aligned} \tag{3.9}$$

At any fixed time, $r = 2n_2$ is the 'bubble-bolt', which is topologically $S^1 \times S^2$. The S^2 factor is described by the (θ_2, ϕ_2) coordinates and it has constant size in time. On the other hand, the circle S^1 described by the ϕ_1 coordinate expands or contracts in time. Again, the first three geometries describe three different evolutions of this circle. The last geometry is static and it is easy to see that it possesses an ergocone with qualitatively the same features as described above for the static bubblenut ergocone.

We can also consider Taub-NUT spaces for which both the 2-dimensional factors M_i are taken to be both a torus T^2 or a hyperboloid H^2. Such geometries and the nutty bubbles obtained from them are presented in appendix A.

Finally, let us notice that all the nutty bubbles geometries exhibited so far have no curvature singularities. Generically, from the form of the metrics one would expect that $r = n_2$ be a curvature singularity. However, the bubblenut solution described above is completely regular at $r = n_2$ as one can check by looking at some of the curvature invariants (for example $R_{\alpha\beta\gamma\delta}R^{\alpha\beta\gamma\delta}$). For the bubble-bolt, this curvature singularity is simply avoided by requiring that $r \geq 2n_2$.

3.2 Bubbles in cosmological backgrounds

Another class of solutions is given for base spaces that are products of 2 -dimensional Einstein manifolds $M_1 \times M_2$. In this case, the metric ansatz that we use to construct the Taub-NUT solution is the one given in (3.2), where now $M = M_1$ while $Y = M_2$.

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As an example we shall consider again the case in which $M_1 = M_2 = S^2$. The metric is written in the form [16]:

$$\begin{aligned} ds^2 &= -F(r)(dt - 2n\cos\theta_1 d\phi_1)^2 + F^{-1}(r)dr^2 \\ &\quad +(r^2+n^2)(d\theta_1^2 + \sin^2\theta_1 d\phi_1^2) + \alpha r^2(d\theta_2^2 + \sin^2\theta_2 d\phi_2^2). \end{aligned} \tag{3.10}$$

In order to satisfy the field equations we must take

$$\alpha = \frac{2}{2-\lambda n^2}, \qquad F(r) = \frac{3r^5 + (l^2+10n^2)r^3 + 3n^2(l^2+5n^2)r - 6ml^2}{3rl^2(r^2+n^2)}.$$

The metric (3.10) is a solution of the vacuum Einstein field equations with cosmological constant $\lambda = -\frac{10}{l^2}$, for any values of n or λ. However, in order retain a metric of Lorentzian signature we must ensure that $\alpha > 0$, which translates in our case to $\lambda n^2 < 2$. For convenience, we have given above the form of $F(r)$ using a negative cosmological constant and in this case the constraint on n and λ is superfluous. We can also use a positive cosmological constant (we have to analytically continue $l \to il$ in $F(r)$) and as long as the above condition on α is satisfied the final metric has Lorentzian signature. The Euclidean section is

$$\begin{aligned} ds^2 &= F_E(r)(d\chi + 2n\cos\theta_1 d\phi_1)^2 + F_E^{-1}(r)dr^2 \\ &\quad +(r^2-n^2)(d\theta_1^2 + \sin^2\theta_1 d\phi_1^2) + \alpha_E r^2(d\theta_2^2 + \sin^2\theta_2 d\phi_2^2) \\ \alpha_E &= \frac{l^2}{l^2-5n^2}, \qquad F_E(r) = \frac{3r^5 + (l^2-10n^2)r^3 - 3n^2(l^2-5n^2)r - 6ml^2}{3rl^2(r^2-n^2)} \end{aligned} \tag{3.11}$$

obtained by continuing $t \to i\chi$ and $n \to in$.

We are now ready to construct the nutty bubbles. First, let us notice that we can analytically continue the coordinates independently in the two S^2 sectors. Let us perform the analytical continuation of one of the coordinates in the second S^2 factor, in which case we obtain the metrics:

$$\begin{aligned} ds^2 &= F_E(r)(d\chi + 2n\cos\theta_1 d\phi_1)^2 + F_E^{-1}(r)dr^2, \\ &\quad +(r^2-n^2)(d\theta_1^2 + \sin^2\theta_1 d\phi_1^2) + \alpha_E r^2(-dt^2 + \cosh^2 t d\phi_2^2), \\ ds^2 &= F_E(r)(d\chi + 2n\cos\theta_1 d\phi_1)^2 + F_E^{-1}(r)dr^2, \\ &\quad +(r^2-n^2)(d\theta_1^2 + \sin^2\theta_1 d\phi_1^2) + \alpha_E r^2(-dt^2 + \sinh^2 t d\phi_2^2), \\ ds^2 &= F_E(r)(d\chi + 2n\cos\theta_1 d\phi_1)^2 + F_E^{-1}(r)dr^2, \\ &\quad +(r^2-n^2)(d\theta_1^2 + \sin^2\theta_1 d\phi_1^2) + \alpha_E r^2(-dt^2 + e^{2t} d\phi_2^2), \\ ds^2 &= F_E(r)(d\chi + 2n\cos\theta_1 d\phi_1)^2 + F_E^{-1}(r)dr^2, \\ &\quad +(r^2-n^2)(d\theta_1^2 + \sin^2\theta_1 d\phi_1^2) + \alpha_E r^2(d\theta_2^2 - \sin^2\theta_2 dt^2). \end{aligned} \tag{3.12}$$

The above metrics are solutions of Einstein field equations with cosmological constant for any values of $\lambda = -10/l^2$ and n. In the case considered here, for a negative cosmological constant, α_E can have negative values if $5n^2 > l^2$. However, while in the Euclidean sector negative values of α_E are not permitted, for our nutty bubbles a negative value for α_E

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amounts to an overall sign change of the metric in the (t,ϕ_2) (respectively (t,θ_2)) sectors. We shall see that this can have a dramatic influence on the dynamical evolution of the bubble.

The bubble will be located at the highest root r_0 of $F_E(r)$ chosen such that $r_0 > n$ and in general we restrict the range of the r coordinate $r \geq r_0$. Elimination of the Misner string sets the periodicity of the χ coordinate to be $8\pi n/k$ and we also have to match it with the periodicity $4\pi/|F_E'(r_0)|$ introduced after we eliminate any possible conical singularities in the (χ, r) sector. Again we have two solutions: a nut and a bolt.

The nut solution corresponds to a two-dimensional fixed-point set of the vector $\frac{\partial}{\partial\chi}$ located at $r = n$. The periodicity of the χ coordinate is in this case equal to $8\pi n$ and the value of the mass parameter is fixed to $m_n = \frac{n^3(4n^2-l^2)}{3l^2}$. Notice that the mass parameter can take any values: positive, negative or zero. There is a curvature singularity at the bubble location! Furthermore, if $n = \frac{l}{2}$ then curvature singularity present at $r = n$ disappears and the spacetime has constant curvature. The fixed-points set of the isometry generated by $\partial/\partial\chi$ is effectively two dimensional. The induced geometry on the 'bubblenut' is a two-dimensional de Sitter space:

$$\begin{aligned}
ds_2^2 &= \alpha_E n^2(-dt^2 + \cosh^2 t d\phi_2^2), \\
ds_2^2 &= \alpha_E n^2(-dt^2 + \sinh^2 t d\phi_2^2), \\
ds_2^2 &= \alpha_E n^2(-dt^2 + e^{2t} d\phi_2^2), \\
ds_2^2 &= \alpha_E n^2(d\theta_2^2 - \sin^2\theta_2 dt^2).
\end{aligned} \tag{3.13}$$

If the coordinate ϕ_2 is periodically identified then at any fixed time $r = n$ is our 'bubblenut': a circle with minimal circumference which expands or contracts. The first three de Sitter geometries above correspond to three different evolutions of this circle as with (2.12). The last geometry is static as in these coordinates the metric has a Killing vector $\xi = \partial/\partial t$.

Now let us consider the effect of changing the sign of α_E. This can be easily accommodated by taking $l^2 < 5n^2$. As we can easily see from the metric induced on the bubble, a negative sign of α_E amounts to changing the induced de Sitter geometry of the bubblenut into a two-dimensional anti-de Sitter geometry. As it is well known, this space can have non-trivial identifications and so it can describe for instance a two-dimensional black hole (as in the second and third metrics above), while the first metric describes pure AdS in standard coordinates. After a coordinate transformation these metrics can be written in the form:

$$\begin{aligned}
ds^2 &= (-\alpha_E)n^2\left(\frac{dx^2}{x^2+k} - (x^2+k)dt^2\right), \\
ds_2^2 &= (-\alpha_E)n^2(-dt^2 + \sin^2 t d\phi_2^2),
\end{aligned} \tag{3.14}$$

where $k = -1, 1, 0$ for the respective first three metrics in (3.13). They are all locally equivalent under changes of coordinates. However, depending on the identifications made, the global structure can be quite different. For example, the second metric from (3.14) can be locally transformed into the $k = -1$ metric by a coordinate transformation. However, if ϕ_2 is periodic then the global structure of these spaces is completely different. For $r \geq n$

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this metric describes a bubble located at $r = n$, which expands from zero size to a finite size $(-\alpha_E n^2)$ and then contracts to zero size again. Near the initial expansion (or the final contraction) the scale factor is linear in time and the spacetime expands or contracts like a Milne universe.

The bubble bolt geometry has a four-dimensional fixed-point set of $\frac{\partial}{\partial\chi}$ located at $r = r_b$ with:

$$r_b = \frac{kl^2 \pm \sqrt{k^2 l^4 - 80n^2 l^2 + 400n^4}}{20n}, \tag{3.15}$$

while the value of the mass parameter is:

$$m_b = \frac{3r_b^5 + (l^2 - 10n^2)r_b^3 - 3n^2(l^2 - 5n^2)r_b}{6l^2}. \tag{3.16}$$

The periodicity of the coordinate χ is $\frac{8\pi n}{k}$, where k is an integer. The induced geometry on the 'bubblebolt' is a two-dimensional de Sitter space times a sphere S^2:

$$\begin{aligned}
ds_2^2 &= (r_b^2 - n^2)(d\theta_1^2 + \sin^2\theta_1 d\phi_1^2) + \alpha_E r_b^2(-dt^2 + \cosh^2 t d\phi_2^2), \\
ds_2^2 &= (r_b^2 - n^2)(d\theta_1^2 + \sin^2\theta_1 d\phi_1^2) + \alpha_E r_b^2(-dt^2 + \sinh^2 t d\phi_2^2), \\
ds_2^2 &= (r_b^2 - n^2)(d\theta_1^2 + \sin^2\theta_1 d\phi_1^2) + \alpha_E r_b^2(-dt^2 + e^{2t} d\phi_2^2), \\
ds_2^2 &= (r_b^2 - n^2)(d\theta_1^2 + \sin^2\theta_1 d\phi_1^2) + \alpha_E r_b^2(d\theta_2^2 - \sin^2\theta_2 dt^2).
\end{aligned} \tag{3.17}$$

At any fixed time, $r = r_b$ is the 'bubblebolt', which is topologically $S^1 \times S^2$. The S^2 factor is described by the (θ_1, ϕ_1) coordinates and it has constant size in time. On the other hand, the circle S^1 described by ϕ_2, expands or contracts in time. Again, the first three geometries describe three different evolutions of this circle. The last geometry in (3.17) is static. As for the bubblenut, changing the sign of α_E has dramatic consequences as it effectively turns the two-dimensional dS geometry into AdS.

The boundary geometry for these bubble spacetimes is given by

$$\begin{aligned}
ds^2 &= 4n^2/l^2(d\tilde{\chi} + \cos\theta_1 d\phi_1)^2 + (d\theta_1^2 + \sin^2\theta_1 d\phi_1^2) + \alpha_E dS_2, \\
ds^2 &= 4n^2/l^2(d\tilde{\chi} + \cos\theta_1 d\phi_1)^2 + (d\theta_1^2 + \sin^2\theta_1 d\phi_1^2) + (-\alpha_E) d\Sigma_2,
\end{aligned} \tag{3.18}$$

where $\tilde{\chi} = \chi/2n$ is periodic with period 4π. Here dS_2 (respectively $d\Sigma_2$) describes the metric of a two-dimensional de Sitter space (respectively AdS). The (χ, θ_1, ϕ_1)-sector describes a squashed three-sphere, the squashing parameter being controlled by $4n^2/l^2$. It is interesting to note that, for negative α_E, one can perform identifications on the AdS part of the metric which turn it into a black-hole.

Finally, the other possibility to obtain bubble spacetimes is to analytically continue $t \to i\chi$ and one of the coordinates in the first S^2 factor in (3.10):

$$\begin{aligned}
ds^2 &= F(r)(d\chi + 2n\sinh t d\phi_1)^2 + F^{-1}(r)dr^2 \\
&\quad + (r^2 + n^2)(-dt^2 + \cosh^2 t d\phi_1^2) + \alpha r^2(d\theta_2^2 + \sin^2\theta_2 d\phi_2^2), \\
ds^2 &= F(r)(d\chi + 2n\cosh t d\phi_1)^2 + F^{-1}(r)dr^2
\end{aligned}$$

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$$
\begin{aligned}
&+(r^2+n^2)(-dt^2+\sinh^2 t d\phi_1^2)+\alpha r^2(d\theta_2^2+\sin^2\theta_2 d\phi_2^2),\\
ds^2 &= F(r)(d\chi+2ne^t d\phi_1)^2+F^{-1}(r)dr^2,\\
&+(r^2+n^2)(-dt^2+e^{2t}d\phi_1^2)+\alpha r^2(d\theta_2^2+\sin^2\theta_2 d\phi_2^2),\\
ds^2 &= F(r)(d\chi+2n\cos\theta_1 dt)^2+F^{-1}(r)dr^2,\\
&+(r^2+n^2)(d\theta_1^2-\sin^2\theta_1 dt^2)+\alpha r^2(d\theta_2^2+\sin^2\theta_2 d\phi_2^2).
\end{aligned}
\tag{3.19}
$$

The above metrics are solutions of vacuum Einstein field equations with cosmological constant for any values of n or $\lambda = -10/l^2$. However, in order to keep the signature of the metric Lorentzian we have to ensure that $\alpha > 0$ i.e. $\lambda n^2 < 2$. We can have a positive[6] or negative cosmological constant as long as this relation is satisfied. Notice that for a negative cosmological constant α is always positive. The bubble is located at the biggest root r_0 of $F(r)$ and in order to eliminate a conical singularity in the (χ, r)-sector, we have to periodically identify χ with period given by $4\pi/|F'(r_0)| = 4\pi l^2 r_0/[l^2 + 5(r_0^2 + n^2)]$. At any fixed time, $r = r_0$ is the bubble, which is topologically $S^1 \times S^2$. The S^2 factor is described by the (θ_2, ϕ_2) coordinates and it has constant size. On the other hand, the circle S^1 described by ϕ_1 expands or contracts in time. Again, the first three geometries describe three different evolutions of this circle. The last geometry is stationary and it is easy to see that it possesses an ergocone with qualitatively the same features as described above for the static bubblenut ergocones encountered in the previous sections.

Similar nutty bubbles can be obtained by considering Taub-NUT metrics for which $M_1 \neq M_2$. Such metrics have been studied in [16, 18, 17]. In six dimensions, such metrics can have two independent nut parameters and there exist a constraint on the values of these nut parameters and the cosmological constant. Quite generically this constraint makes it impossible to set the cosmological constant to zero. Having two nut parameters at our disposal it is very easy to construct various nutty bubble solutions by analytically continuing the coordinates in only one of the factors M_i. We exhibit in appendix A more examples of such geometries.

3.3 Nutty bubbles and Hopf dualities

We shall now briefly describe a method to generate new time-dependent solutions starting from some of the nutty bubbles studied in the previous sections. This method was based on the fact that, in general, the odd-dimensional spheres S^{2n+1} may be regarded as circle bundles over CP^n and one can use the so-called Hopf duality (a T-duality along the U(1)-fibre) to generate new solutions by untwisting S^{2n+1} to $CP^n \times S^1$, as in [19 − 22]. The six-dimensional case is particularly interesting for us since it has been shown in [20] that it is possible to make consistent truncations of the maximal Type II supergravity theories to a bosonic sector which exhibits an $O(2, 2)$ global symmetry, with the T-duality transformation taking a very simple form. The theories at hand are the toroidal reductions of Type IIA, respectively Type IIB ten-dimensional supergravities, while the reduction ansatz for the fields is that the six-dimensional fields that are retained are precisely the ten-dimensional ones, with the spacetime indices restricted to run over the six-dimensional

[6] To write the solution for a positive cosmological constant we have to send $l \to il$ in $F(r)$ in (3.2).

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range only. The two truncated theories in $D = 6$ are then related by a T-duality transformation upon reduction to $D = 5$. The explicit mappings of the fields have been given in [20] and we mainly follow their notational conventions. However, for convenience we also provide the derivation of the T-duality rules in appendix B.

In this section we apply Hopf dualities to some of our nutty bubbles to generate new time dependent backgrounds. However, since these rules work only for solutions that do not have a cosmological constant, we shall focus mainly on the cases in which $M_1 = M_2$. For simplicity, and just to illustrate the method we shall use just a few nutty bubbles solutions as seeds.

Let us start with the solution given in (3.6). Considering this metric as a solution of the pure gravity sector of the truncated Type IIA theory we can now perform a Hopf-duality along the spacelike χ -direction to obtain a solution of six-dimensional Type IIB theory:

$$\begin{aligned} ds_{6B} &= \tilde{F}_E(r)^{-\frac{1}{2}}d\chi^2 + \tilde{F}_E(r)^{-\frac{1}{2}}dr^2 + \tilde{F}_E(r)^{\frac{1}{2}}(r^2+n_1^2)(-dt^2+\cosh^2 t d\phi_1^2) \\ &\quad +\tilde{F}_E(r)^{\frac{1}{2}}(r^2-n_2^2)(d\theta_2^2+\sin^2\theta_2 d\phi_2^2)^2 \\ e^{2\varphi_1} &= e^{2\varphi_2} = \tilde{F}_E(r), \qquad A_{(2)}^{NS} = 2n_1 \sinh t d\phi_1 \wedge d\chi + 2n_2 \cos\theta_2 d\phi_2 \wedge d\chi. \end{aligned} \tag{3.20}$$

Were we to consider (3.6) as a solution of the pure gravity sector of Type IIB theory, then after performing the spacelike Hopf dualisation we would obtain a solution of Type IIA theory:

$$\begin{aligned} ds_{6A} &= \tilde{F}(r)^{-\frac{1}{2}}d\chi^2 + \tilde{F}(r)^{-\frac{1}{2}}dr^2 + \bar{F}(r)^{\frac{1}{2}}(r^2+n_1^2)(-dt^2+\cosh^2 t d\phi_1^2) \\ &\quad +\bar{F}(r)^{\frac{1}{2}}(r^2-n_2^2)(d\theta_2^2+\sin^2\theta_2 d\phi_2^2) \\ e^{2\varphi_1} &= e^{2\varphi_2} = \bar{F}(r), \qquad A_{(2)} = 2n_1 \sinh t d\phi_1 \wedge d\chi + 2n_2 \cos\theta_2 d\phi_2 \wedge d\chi. \end{aligned} \tag{3.21}$$

The analysis of these charged bubbles proceeds as in the previous sections. The bubble will be located at the largest root of $\tilde{F}_E(r)$. Generically there exists a curvature singularity at the bubble location, which cannot be cured by any appropriate choices of the parameters. Another difference with the previous bubble solutions is that in the (χ, r) -sector there is no conical singularity to be eliminated and χ need not be compactified.

As another example of this method, let us consider a bubble solution, given in appendix A, which corresponds to a six-dimensional Taub-NUT constructed as a circle fibration over $T^2 \times T^2$. Taking the bubble solution given in (A.5) as a solution of Type IIA theory, then after performing a Hopf duality along the χ direction we obtain:

$$\begin{aligned} ds_{6B} &= \tilde{F}_E(r)^{-\frac{1}{2}}d\chi^2 + \tilde{F}_E(r)^{-\frac{1}{2}}dr^2 + \tilde{F}_E(r)^{\frac{1}{2}}(r^2+n_1^2)(-dt^2+d\phi_1^2) \\ &\quad +\tilde{F}_E(r)^{\frac{1}{2}}(r^2-n_2^2)(d\theta_2^2+d\phi_2^2) \\ e^{2\varphi_1} &= e^{2\varphi_2} = \tilde{F}_E(r), \qquad A_{(2)}^{NS} = 2n_1 t d\phi_1 \wedge d\chi + 2n_2 \theta_2 d\phi_2 \wedge d\chi. \end{aligned} \tag{3.22}$$

which is a solution of the Type IIB theory. Notice that $F(r) = 0$ only if $r = 0$. On the other hand, we have to restrict the values of the radial coordinates such that $r \geq n_2$, or else the signature of this metric will change. There is however a curvature singularity at $r = n_2$, which cannot be eliminated by any appropriate choices of the parameter m.

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4. Discussion

In this paper, we have constructed a wide variety of time-dependent backgrounds using the standard techniques of analytic continuation. Since many of the presented solutions are locally asymptotically (A)dS, they are relevant in the context of gauge/gravity dualities.

For example, let us discuss one of our solutions (2.12) in the context of the AdS/CFT correspondence. The bulk-boundary correspondence in the Lorentzian section demands the inclusion of both normalizable and non-normalizable modes of the bulk fields [29]. The former propagate in the bulk and correspond to physical states while the latter serve as classical, non-fluctuating backgrounds and encode the choice of operator insertions in the boundary theory.

Since the bulk theory is a theory of gravity, one of the bulk fields will always be the graviton (metric perturbations). The AdS/CFT dictionary tells us that its dual operator is the stress-energy tensor of the CFT. We will compare the dual CFT stress tensor to the *rescaled* boundary stress tensor calculated from the bulk spacetime using the counterterm subtraction procedure of [30, 31]. Typically, the boundary of a locally asymptotically spacetime will be an asymptotic surface at some large radius r. However, the metric restricted to the boundary γ_{ab} diverges due to an infinite conformal factor r^2/ℓ^2, and so the metric upon which the dual field theory resides is usually defined using the rescaling

$$h_{ab} = \lim_{r\to\infty} \frac{\ell^2}{r^2}\gamma_{ab}. \tag{4.1}$$

Corresponding to the boundary metric h_{ab}, the stress-energy tensor $\langle\tau_{ab}\rangle$ for the dual theory can be calculated using the following relation

$$\sqrt{-h}h^{ab}\langle\tau_{bc}\rangle = \lim_{r\to\infty} \sqrt{-\gamma}\gamma^{ab}T_{bc}. \tag{4.2}$$

In our case, the boundary metric is

$$ds^2 = h_{ab}dx^a dx^b = (d\chi + lxdt)^2 + l^2\left(\frac{dx^2}{x^2+k} - (x^2+k)dt^2\right) + l^2dy^2, \tag{4.3}$$

and so the conformal boundary, where the $\mathcal{N}=4$ SYM lives, is $AdS_3 \times S^1$. The *rescaled* boundary stress tensor is

$$\begin{aligned}
\tau_t^t &= \frac{256m + 5l^2}{1024\pi G l^3}, \\
\tau_\chi^t &= 0, \\
\tau_t^\chi &= -\frac{(l^2+64m)x}{64\pi G l^3} \\
\tau_\chi^\chi &= -\frac{11l^2 + 768m}{1024\pi G l^3}, \\
\tau_x^x &= \frac{5l^2 + 256m}{1024\pi G l^3}, \\
\tau_y^y &= \frac{l^2 + 256m}{1024\pi G l^3}.
\end{aligned} \tag{4.4}$$

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Since the boundary metric is the product of a circle and a three-dimensional Einstein space, the trace anomaly vanishes. Indeed, as we expected, the stress tensor (4.4) is finite, covariantly conserved, and manifestly traceless.

For four dimensions, it was shown in [5] that in the special case when the NUT charge vanishes ($n = 0$), the metric (stress tensor) reduces to the 4-dimensional Schwarzschild-AdS metric (stress tensor). In five dimensions, the constraint between the NUT charge and the cosmological constant changes dramatically the situation — we will comment on this at the end of this section. However, the limit we are interested here is $m = -64/l^2$. Then, the bulk geometry has constant curvature and it is the static bubble obtained from AdS_5 by analytical continuation. Indeed, in this case $F(r) = \frac{r^2}{l^2} + \frac{1}{4}$ and by redefining the coordinate $r^2 \to r^2 - \frac{l^2}{4}$ and rescaling y to absorb an l^2 factor, the metric can be cast in the form:

$$ds^2 = \left(\frac{r^2}{l^2} - \frac{1}{4}\right) dy^2 + \frac{dr^2}{\left(\frac{r^2}{l^2} - \frac{1}{4}\right)} + r^2[(d\tilde{\chi} + \sinh\theta dt)^2 + d\theta^2 - \cosh^2\theta dt^2].$$

One can recognize it as being the analytic continuation of AdS_5 with a non-canonically normalized H^3 factor. For this particular value of the parameter m, the stress tensor (4.4) becomes

$$\tau^a_b = \frac{N^2}{512\pi^2 l^4}\, \mathrm{diag}(1, 1, 1, -3), \tag{4.5}$$

where we have used the standard relation $l^3/G = 2N^2/\pi$ to rewrite the stress tensor in terms of field theory quantities.[7]

Let us move now to the dual theory that is in terms of $\mathcal{N} = 4$ SYM on the $AdS_3 \times S^1$ spacetime. This is a conformally flat spacetime and, fortunately, there is a standard result for the stress tensor [32]:

$$\langle\tau^a_b\rangle = -\frac{1}{16\pi^2}\left(A^{(1)}H^a_b + B^{(3)}H^a_b\right) + \tilde{\tau}^a_b. \tag{4.6}$$

Here, ${}^{(1)}H^a_b$ and ${}^{(3)}H^a_b$ are conserved quantities constructed from the curvature (see [32] for their definitions), and $\tilde{\tau}^a_b$ is a traceless state-dependent part. In our case they are given by

$$\begin{aligned} {}^{(1)}H^a_b &= \frac{3}{8l^4}\mathrm{diag}[1, 1, 1, -3], \\ {}^{(3)}H^a_b &= -\frac{1}{16l^4}\mathrm{diag}[1, 1, 1, -3]. \end{aligned} \tag{4.7}$$

The coefficients A and B are calculated as in [4]. The trace of (4.6) is compared with the conformal anomaly for $\mathcal{N} = 4$ SYM [31]:

$$\langle\tau^a_a\rangle = -\frac{1}{16\pi^2}\left(-6A\Box R - B(R_{ab}R^{ab} - 1/3R^2)\right) = \frac{(N^2-1)}{64\pi^2}(2R_{ab}R^{ab} - 2/3R^2). \tag{4.8}$$

[7] The convention for the coordinates is $1, 2, 3, 4 = t, \chi, x, y$.

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This fixes $A = 0$ and $B = (N^2 - 1)/2$ and so the field theory stress tensor becomes

$$\langle \tau^a_b \rangle = \frac{1}{2} \frac{(N^2-1)}{256\pi^2 l^4} \,\mathrm{diag}(1,1,1,-3) + \tilde{\tau}^a_b. \tag{4.9}$$

In the large N limit, the geometrical part of the stress tensor precisely reproduces (4.5). The fact that the geometrical part is non zero is a direct consequence of analytic continuation — the quantum field theory on the AdS boundary can have a nonvanishing vacuum (Casimir) energy. Consequently, the above comparison of the stress tensor (4.5) to (4.9) does result in a non-trivial connection between them.

As advertised in section 2, we would like to comment here on the limit when the NUT charge and/or the cosmological constant are zero. Central to our construction was starting with an appropriate NUT-charged family of (generating) solutions in higher dimensions. These solutions can have more than one nut parameter. Since in higher dimensions there is a constraint between the nut parameters and the cosmological constant, the limit mentioned above is more subtle than in four dimensions. Of course, these remarks apply to odd dimensions, particularly to the 5-dimensional solutions in section 2. However, as noted there, there is a way to evade this situation: after a change of coordinates and by performing appropriate rescalings, it turns out that that the metric can be cast in such a form that allows us to take the limit of a vanishing cosmological constant.[8]

More precisely, start with the metric

$$ds^2 = F_E(r)(d\chi - l\cos\theta d\varphi)^2 + F_E^{-1}(r)dr^2 + \left(r^2 - \frac{l^2}{4}\right)(d\theta^2 + \sin^2\theta d\varphi^2) + r^2 dz^2, \tag{4.10}$$

where

$$F_E(r) = \frac{4r^4 - 2l^2r^2 + 16ml^2}{l^2(4r^2 - l^2)}. \tag{4.11}$$

Make now the coordinate change $r^2 \to r^2 + l^2/4$ and define $a^4 = 64ml^2 - l^4$. Then, after a further rescaling of the r and y coordinates the metric becomes

$$\begin{aligned} ds^2 =& \frac{r^2}{4}\left(1 - \frac{a^4}{r^4}\right)(d\tilde{\chi} + \cos\theta d\phi)^2 + \frac{dr^2}{\left(\frac{r^2}{l^2} - 1\right)\left(1 - \frac{a^4}{r^4}\right)} \\ &+ \frac{r^2}{4}(d\theta^2 + \sin^2\theta d\phi^2) + \left(\frac{r^2}{l^2} - 1\right) d\tilde{y}^2, \end{aligned} \tag{4.12}$$

which is a solution of the 5-dimensional Einstein equations with cosmological constant, referred to as the Eguchi-Hanson soliton [33]. The limit in which the cosmological constant vanishes is now a smooth limit and the metric becomes the product of the four-dimensional Eguchi-Hanson metric with a trivial flat direction.

There also exists a limit in which we can set the nut parameter to zero. Namely, our 5-dimensional metric can be written quite generally in the form [17]

$$ds^2 = -F(r)(dt + 2\frac{n}{\delta}\cos\theta d\phi)^2 + \frac{dr^2}{F(r)} + \frac{r^2 + n^2}{\delta}(d\theta^2 + \sin^2\theta d\phi^2) + r^2 dy^2,$$

[8] The metric obtained this way is a generalization of the Eguchi-Hanson metric in higher dimensions [33] (see, also, [17]).

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$$F(r) = \frac{r^4 + 2n^2 r^2 - 2ml^2}{r^2 + n^2}, \tag{4.13}$$

and the constrain equation is now simply $\delta = -\frac{4n^2}{l^2}$. This metric is then a solution of Einstein field equations with cosmological constant $\Lambda = -\frac{6}{l^2}$. Once we fix δ as above, there is no constraint on the values of Λ and n other than the requirement of a metric of Lorentzian signature — this can be easily accommodated by analytically continuing the coordinate $\theta \to i\theta$. Defining now a new nut parameter $N = \frac{n}{\delta}$ and $\lambda = -\frac{4}{l^2}$, the above solution can be written in the following form:

$$\begin{aligned} ds^2 &= -F(r)(dt - 2N\cosh\theta d\phi)^2 + \frac{dr^2}{F(r)} + \frac{\lambda^2 N^2 r^2 + 1}{(-\lambda)}(d\theta^2 + \sinh^2\theta d\phi^2) + r^2 dy^2, \\ F(r) &= \frac{16N^2 r^4 + 2r^2 l^2 - ml^2}{l^2(16N^2 r^2 + l^4)}. \end{aligned} \tag{4.14}$$

When $N \neq 0$, a change of coordinates will bring the metric into a form similar to the one discussed in section 2. Notice however that, in the form written above, it is possible to take a smooth limit of the metric in which the nut charge $N \to 0$. Then, we obtain a metric that is the trivial product of a 3-dimensional Schwarzschild AdS (described by the coordinates (t, r, y)) with a 2-dimensional hyperboloid (described by (θ, ϕ)).

Finally, we note that the boundary of the solutions presented in the paper is generically a circle-fibration over base spaces — it is obtained from products of general Einstein-Kähler manifolds and can have exotic topologies. Then, one should be able to understand the thermodynamic phase structure of the dual field theory by working out the corresponding phase structure of our gravity solutions in the bulk. We leave a more detailed study of these solutions and their duals for future work.

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A. Other six-dimensional bubble spacetimes

U^1-fibration over $S^2 \times T^2$ Let us consider now the case of a Taub-NUT space that appears as a radial extension of a U(1)-fibration over $S^2 \times T^2$ in the Euclidean sector [16]:

$$\begin{aligned} ds^2 &= F_E(r)(d\chi + 2n_1\cos\theta_1 d\phi_1 + 2n_2\theta_2 d\phi_2)^2 + F_E^{-1}(r)dr^2 \\ &\quad + (r^2 - n_1^2)(d\theta_1^2 + \sin^2\theta_1 d\phi_1^2) + (r^2 - n_2^2)(d\theta_2^2 + d\phi_2^2) \\ F_E(r) &= \frac{3r^6 + (l^2 - 5n_2^2 - 10n_1^2)r^4 + 3(-n_2^2 l^2 + 10n_1^2 n_2^2 - n_1^2 l^2 + 5n_1^4)r^2}{3(r^2 - n_1^2)(r^2 - n_2^2)l^2} \\ &\quad + \frac{6ml^2 r - 3n_1^2 n_2^2(l^2 - 5n_1^2)}{3(r^2 - n_1^2)(r^2 - n_2^2)l^2} \end{aligned} \tag{A.1}$$

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The above metric will be solution of the Einstein field equations with cosmological constant if and only if we have $(n_2^2 - n_1^2)\lambda = 2$. As a consequence we are forced to consider a non-zero cosmological constant. We have now two possibilities: we can perform analytic continuations in the S^2 sector or we can do that in the T^2 sector. In the first case we make analytic continuations in the S^2 sector, which in turn forces us to take $n_1 \to in_1$:

$$\begin{aligned}
ds^2 &= \tilde{F}_E(r)(d\chi + 2n_1 \sinh t d\phi_1 + 2n_2\theta_2 d\phi_2)^2 + \tilde{F}_E^{-1}(r)dr^2 \\
&\quad +(r^2+n_1^2)(-dt^2 + \cosh^2 t d\phi_1^2) + (r^2 - n_2^2)(d\theta_2^2 + d\phi_2^2) \\
ds^2 &= \tilde{F}_E(r)(d\chi + 2n_1 \cosh t d\phi_1 + 2n_2\theta_2 d\phi_2)^2 + \tilde{F}_E^{-1}(r)dr^2 \\
&\quad +(r^2+n_1^2)(-dt^2 + \sinh^2 t d\phi_1^2) + (r^2 - n_2^2)(d\theta_2^2 + d\phi_2^2) \\
ds^2 &= \tilde{F}_E(r)(d\chi + 2n_1 e^t d\phi_1 + 2n_2\theta_2 d\phi_2)^2 + \tilde{F}_E^{-1}(r)dr^2 \\
&\quad +(r^2+n_1^2)(-dt^2 + e^{2t} d\phi_1^2) + (r^2 - n_2^2)(d\theta_2^2 + d\phi_2^2) \\
ds^2 &= \tilde{F}_E(r)(d\chi + 2n_1 \cos\theta_1 dt + 2n_2\theta_2 d\phi_2)^2 + \tilde{F}_E^{-1}(r)dr^2 \\
&\quad +(r^2+n_1^2)(d\theta_1^2 - \sin^2\theta_1 dt^2) + (r^2 - n_2^2)(d\theta_2^2 + d\phi_2^2) \\
\tilde{F}_E(r) &= -\frac{3r^6 + (-l^2 - 5n_2^2 + 10n_1^2)r^4 + 3(n_2^2 l^2 - 10n_1^2 n_2^2 - n_1^2 l^2 + 5n_1^4)r^2}{3(r^2+n_1^2)(r^2-n_2^2)l^2} \\
&\quad + \frac{6ml^2 r + 3n_1^2 n_2^2 (l^2 - 5n_1^2)}{3(r^2+n_1^2)(r^2-n_2^2)l^2}
\end{aligned} \tag{A.2}$$

The above metrics will be solutions of the Einstein field equations with cosmological constant if and only if $(n_1^2 + n_2^2)\lambda = 2$. Hence we are also constrained to have only a positive cosmological constant which implies that our solutions are time-dependent asymptotically de Sitter spaces (notice that, in view of this fact we have already continued $l \to il$ in the above expression for $\tilde{F}_E$). On the other hand, if we perform analytic continuations in the T^2 sector we are forced to take $n_2 \to in_2$ and we obtain the metrics:

$$\begin{aligned}
ds^2 &= \bar{F}_E(r)(d\chi + 2n_1 \cos\theta_1 d\phi_1 + 2n_2 t d\phi_2)^2 + \bar{F}_E^{-1}(r)dr^2 \\
&\quad +(r^2-n_1^2)(d\theta_1^2 + \sin^2\theta_1 d\phi_1^2) + (r^2 + n_2^2)(-dt^2 + d\phi_2^2) \\
ds^2 &= \bar{F}_E(r)(d\chi + 2n_1 \cos\theta_1 d\phi_1 + 2n_2\theta_2 dt)^2 + \bar{F}_E^{-1}(r)dr^2 \\
&\quad +(r^2-n_1^2)(d\theta_1^2 + \sin^2\theta_1 d\phi_1^2) + (r^2 + n_2^2)(d\theta_2^2 - dt^2) \\
\bar{F}_E(r) &= \frac{3r^6 + (l^2 + 5n_2^2 - 10n_1^2)r^4 + 3(n_2^2 l^2 - 10n_1^2 n_2^2 - n_1^2 l^2 + 5n_1^4)r^2}{3(r^2-n_1^2)(r^2+n_2^2)l^2} \\
&\quad + \frac{6ml^2 r + 3n_1^2 n_2^2 (l^2 - 5n_1^2)}{3(r^2-n_1^2)(r^2+n_2^2)l^2}
\end{aligned} \tag{A.3}$$

The above metrics will be solutions of Einstein field equations if and only if $(n_1^2+n_2^2)\lambda = -1$, which means that the cosmological constant must be negative. In conclusions the above solutions are time-dependent backgrounds that are asymptotically AdS.

U(1)-fibrations over $T^2 \times T^2$ Let us consider next the analytic continuation of the Taub-NUT spaces that appear as U(1)-fibrations over $T^2 \times T^2$. The Euclidean version of such spaces is [16]:

$$ds^2 = F_E(r)(d\chi + 2n_1\theta_1 d\phi_1 + 2n_2\theta_2 d\phi_2)^2 + F_E^{-1}(r)dr^2$$

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$$
\begin{aligned}
&+(r^2-n_1^2)(d\theta_1^2+d\phi_1^2)+(r^2-n_2^2)(d\theta_2^2+d\phi_2^2)\\
F_E(r) &= \frac{3r^6-5(n_2^2+2n_1^2)r^4+15n_1^2(n_1^2+2n_2^2)r^2+6ml^2r+15n_1^4n_2^2}{3(r^2-n_1^2)(r^2-n_2^2)l^2} \qquad (A.4)
\end{aligned}
$$

The above metric is a solution of vacuum Einstein field equations with cosmological constant if and only if $(n_2^2-n_1^2)\lambda=0$. Hence in the case of a vanishing cosmological constant we can have two independent NUT charges in the metric. We can similarly analytically continue the coordinates from one factor space T^2 only:

$$
\begin{aligned}
ds^2 &= \tilde{F}_E(r)(d\chi+2n_1td\phi_1+2n_2\theta_2d\phi_2)^2+\tilde{F}_E^{-1}(r)dr^2\\
&\quad +(r^2+n_1^2)(-dt^2+d\phi_1^2)+(r^2-n_2^2)(d\theta_2^2+d\phi_2^2)\\
ds^2 &= \tilde{F}_E(r)(d\chi+2n_1\theta_1dt+2n_2\theta_2d\phi_2)^2+\tilde{F}_E^{-1}(r)dr^2\\
&\quad +(r^2+n_1^2)(d\theta_1^2-dt^2)+(r^2-n_2^2)(d\theta_2^2+d\phi_2^2)\\
\tilde{F}_E(r) &= \frac{2mr}{(r^2+n_1^2)(r^2-n_2^2)} \qquad (A.5)
\end{aligned}
$$

However, if $\lambda\neq 0$ we are forced to have $n_1=n_2$ and it is impossible to analytically continue the coordinates of the T^2 factors separately.

A.1 Warped products of nutty spaces

These Taub-NUT spaces have the base space factorized as a product of the form $M_2^{(1)}\times M_2^{(2)}$ where the factors $M_2^{(i)}$ are two-dimensional Einstein spaces with constant curvature. We shall choose them to be of the form S^2, T^2 or H^2. Consider now the case of a warped product of a circle fibration over the S^2 factor of the base space $S^2\times T^2$. The Euclidean version of the metric of these spaces is given by:

$$
\begin{aligned}
ds^2 &= F_E(r)(d\chi+2n\cos\theta_1d\phi_1)^2+F_E^{-1}(r)dr^2+(r^2-n^2)(d\theta_1^2+\sin^2\theta_1d\phi_1^2)\\
&\quad +r^2(d\theta_2^2+d\phi_2^2)\\
F_E(r) &= \frac{3r^5+(l^2+10n^2)r^3-3n^2(l^2+5n^2)r+6ml^2}{3rl^2(r^2-n^2)} \qquad (A.6)
\end{aligned}
$$

The above metric is a solution of vacuum Einstein field equations with a cosmological constant if and only if $\lambda n^2=-2$, i.e. we are constrained to have a negative cosmological constant $\lambda=-\frac{10}{l^2}$. Performing analytic continuations in the S^2 sector we are forced to take $n\to in$ and the constraint equation will become in this case $\lambda n^2=2$. Hence our solutions will be asymptotically de Sitter:

$$
\begin{aligned}
ds^2 &= \tilde{F}_E(r)(d\chi+2n\sinh td\phi_1)^2+\tilde{F}_E^{-1}(r)dr^2+(r^2+n^2)(-dt^2+\cosh^2td\phi_1^2)\\
&\quad +r^2(d\theta_2^2+d\phi_2^2)\\
ds^2 &= \tilde{F}_E(r)(d\chi+2n\cosh td\phi_1)^2+\tilde{F}_E^{-1}(r)dr^2+(r^2+n^2)(-dt^2+\sinh^2td\phi_1^2)\\
&\quad +r^2(d\theta_2^2+d\phi_2^2)\\
ds^2 &= \tilde{F}_E(r)(d\chi+2ne^td\phi_1)^2+\tilde{F}_E^{-1}(r)dr^2+(r^2+n^2)(-dt^2+e^{2t}d\phi_1^2)+r^2(d\theta_2^2+d\phi_2^2)\\
ds^2 &= \tilde{F}_E(r)(d\chi+2n\cos\theta_1dt)^2+\tilde{F}_E^{-1}(r)dr^2+(r^2+n^2)(d\theta_1^2-\sin^2\theta_1dt^2)+r^2(d\theta_2^2+d\phi_2^2)\\
\tilde{F}_E &= \frac{-3r^5+(l^2-10n^2)r^3+3n^2(l^2-5n^2)r+6ml^2}{3rl^2(r^2+n^2)} \qquad (A.7)
\end{aligned}
$$

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If we analytically continue the coordinates in the second sector T^2 we obtain:

$$ds^2 = F_E(r)(d\chi + 2n\cos\theta_1 d\phi_1)^2 + F_E^{-1}(r)dr^2 + (r^2 - n^2)(d\theta_1^2 + \sin^2\theta_1 d\phi_1^2) + r^2(d\theta_2^2 - dt^2)$$

We now consider partial fibrations over S^2 with the base space of the form $S^2 \times H^2$. The Euclidean version of these metrics is given by:

$$\begin{aligned} ds^2 &= F_E(r)(d\chi + 2n\cos\theta_1 d\phi_1)^2 + F_E^{-1}(r)dr^2 + (r^2 - n^2)(d\theta_1^2 + \sin^2\theta_1 d\phi_1^2) \\ &\quad + \alpha r^2(d\theta_2^2 + \sinh^2\theta_2 d\phi_2^2) \\ \alpha &= -\frac{2}{\lambda n^2 + 2}, \qquad F_E(r) = \frac{3r^5 + (l^2 - 10n^2)r^3 - 3n^2(l^2 + 5n^2)r + 6ml^2}{3rl^2(r^2 - n^2)} \end{aligned} \tag{A.8}$$

Since α has to be positive, this means that $\lambda n^2 < -2$, therefore the space is asymptotically anti-de Sitter. If we perform analytic continuation in the S^2 sector we obtain:

$$\begin{aligned} ds^2 &= \tilde{F}_E(r)(d\chi + 2n\sinh t d\phi_1)^2 + \tilde{F}_E^{-1}(r)dr^+(r^2 + n^2)(-dt^2 + \cosh^2 t d\phi_1^2) \\ &\quad + \tilde{\alpha} r^2(d\theta_2^2 + \sinh^2\theta_2 d\phi_2^2) \\ ds^2 &= \tilde{F}_E(r)(d\chi + 2n\cosh t d\phi_1)^2 + \tilde{F}_E^{-1}(r)dr^2 + (r^2 + n^2)(-dt^2 + \sinh^2 t d\phi_1^2) \\ &\quad + \tilde{\alpha} r^2(d\theta_2^2 + \sinh^2\theta_2 d\phi_2^2) \\ ds^2 &= \tilde{F}_E(r)(d\chi + 2ne^t d\phi_1)^2 + \tilde{F}_E^{-1}(r)dr^2 + (r^2 + n^2)(-dt^2 + e^{2t} d\phi_1^2) \\ &\quad + \tilde{\alpha} r^2(d\theta_2^2 + \sinh^2\theta_2 d\phi_2^2) \\ ds^2 &= \tilde{F}_E(r)(d\chi + 2n\cos\theta_1 dt)^2 + \tilde{F}_E^{-1}(r)dr^2 + (r^2 + n^2)(d\theta_1^2 - \sin^2\theta_1 dt^2) \\ &\quad + \tilde{\alpha} r^2(d\theta_2^2 + \sinh^2\theta_2 d\phi_2^2) \end{aligned} \tag{A.9}$$

where

$$\tilde{\alpha} = \frac{2}{\lambda n^2 - 2}, \qquad \tilde{F}_E(r) = \frac{-3r^5 + (l^2 + 10n^2)r^3 - 3n^2(l^2 + 5n^2)r + 6ml^2}{3rl^2(r^2 - n^2)}$$

If we perform analytic continuations in the H^2 section of the base space we obtain:

$$\begin{aligned} ds^2 &= F_E(r)(d\chi + 2n\cos\theta_1 d\phi_1)^2 + F_E^{-1}(r)dr^2 \\ &\quad + (r^2 - n^2)(d\theta_1^2 + \sin^2\theta_1 d\phi_1^2) + \alpha r^2(d\theta_2^2 - \cosh^2\theta_2 dt^2) \\ ds^2 &= F_E(r)(d\chi + 2n\cos\theta_1 d\phi_1)^2 + F_E^{-1}(r)dr^2 \\ &\quad + (r^2 - n^2)(d\theta_1^2 + \sin^2\theta_1 d\phi_1^2) + \alpha r^2(d\theta_2^2 - \sinh^2\theta_2 dt^2) \\ ds^2 &= F_E(r)(d\chi + 2n\cos\theta_1 d\phi_1)^2 + F_E^{-1}(r)dr^2 \\ &\quad + (r^2 - n^2)(d\theta_1^2 + \sin^2\theta_1 d\phi_1^2) + \alpha r^2(d\theta_2^2 - e^{2\theta_2} dt^2) \\ ds^2 &= F_E(r)(d\chi + 2n\cos\theta_1 d\phi_1)^2 + F_E^{-1}(r)dr^2 \\ &\quad + (r^2 - n^2)(d\theta_1^2 + \sin^2\theta_1 d\phi_1^2) + \alpha r^2(-dt^2 + \sin^2 t d\phi_2^2) \end{aligned} \tag{A.10}$$

with the same expressions for $F_E(r)$ and β as in the Euclidean version.

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B. T-duality in six dimensions

The Lagrangian in $D = 6$ obtained by dimensional reduction of Type IIB on a torus and after performing a consistent truncation is given by [20]:

$$\begin{aligned}\mathcal{L}_{6B} = & \, eR - \frac{1}{2}e(\partial\varphi_1)^2 - \frac{1}{2}e(\partial\varphi_2)^2 - \frac{1}{2}ee^{2\varphi_1}(\partial\chi_1)^2 - \frac{1}{2}ee^{2\varphi_2}(\partial\chi_2)^2 \\ & -\frac{1}{12}ee^{-\varphi_1-\varphi_2}(F_{(3)}^{NS})^2 - \frac{1}{12}ee^{\varphi_1-\varphi_2}(F_{(3)}^{RR})^2 + \chi_2 dA_{(2)}^{NS} \wedge dA_{(2)}^{RR} \end{aligned} \tag{B.1}$$

where $F_{(3)}^{NS} = dA_{(2)}^{NS}$ and $F_{(3)}^{RR} = dA_{(2)}^{RR} + \chi_1 dA_{(2)}^{NS}$. This Lagrangian is related by T-duality in $D = 5$ to a different six-dimensional theory obtained by making a consistent truncation of Type IIA compactified on a four-dimensional torus. The corresponding Lagrangian is given by:

$$\begin{aligned}\mathcal{L}_{6A} = & \, eR - \frac{1}{2}e(\partial\varphi_1)^2 - \frac{1}{2}e(\partial\varphi_2)^2 - \frac{1}{48}ee^{\frac{\varphi_1}{2}-\frac{3\varphi_2}{2}}(F_{(4)})^2 - \frac{1}{12}ee^{-\varphi_1-\varphi_2}(F_{(3)})^2 \\ & -\frac{1}{4}ee^{\frac{3\varphi_1}{2}-\frac{\varphi_2}{2}}(F_{(2)})^2 \end{aligned} \tag{B.2}$$

where $F_{(4)} = dA_{(3)} - dA_{(2)} \wedge A_{(1)}$, $F_{(3)} = dA_{(2)}$ corresponds to the NS-NS 3-form $F_{(3)1}$ and $F_{(2)} = dA_{(1)}$ is the RR 2-form $\mathcal{F}_{(2)}^1$, with the index '1' denoting here the first reduction step from $D = 11$ to $D = 10$.

Let us focus on Type IIA theory first. Under a dimensional reduction using the formulae from the previous appendix we have:

$$ds_6^2 = e^{\frac{\varphi}{\sqrt{6}}}ds_5^2 + e^{\frac{-3\varphi}{\sqrt{6}}}(dz + \mathcal{A}_{(1)})^2 \tag{B.3}$$

and we obtain the following 5-dimensional Lagrangian:

$$\begin{aligned}\mathcal{L}_{5A} = & \, eR - \frac{1}{2}e(\partial\varphi_1)^2 - \frac{1}{2}e(\partial\varphi_2)^2 - \frac{1}{2}e(\partial\varphi)^2 - \frac{1}{48}ee^{-\frac{3\varphi}{\sqrt{6}}+\frac{\varphi_1}{2}-\frac{3\varphi_2}{2}}(F'_{(4)})^2 \\ & -\frac{1}{12}ee^{\frac{\varphi}{\sqrt{6}}+\frac{\varphi_1}{2}-\frac{3\varphi_2}{2}}(F_{(3)1})^2 - \frac{1}{12}ee^{-\frac{2\varphi}{\sqrt{6}}-\varphi_1-\varphi_2}(F'_{(3)})^2 - \frac{1}{2}ee^{-\frac{4\varphi}{\sqrt{6}}}\mathcal{F}_{(2)}^2 \\ & -\frac{1}{4}ee^{-\frac{\varphi}{\sqrt{6}}+\frac{3\varphi_1}{2}-\frac{\varphi_2}{2}}(F'_{(2)})^2 - \frac{1}{4}ee^{\frac{2\varphi}{\sqrt{6}}-\varphi_1-\varphi_2}(F_{(2)1})^2 - \frac{1}{2}ee^{\frac{3\varphi}{\sqrt{6}}+\frac{3\varphi_1}{2}-\frac{\varphi_2}{2}}(dA_{(0)1})^2 \end{aligned} \tag{B.4}$$

where the field strengths are defined as follows:

$$\begin{aligned} F'_{(2)} &= dA_{(1)} - dA_{(0)1} \wedge \mathcal{A}_{(0)}, \qquad F'_{(3)} = dA_{(2)} - dA_{(1)} \wedge \mathcal{A}_{(1)} \\ F_{(3)1} &= dA_{(2)1} + dA_{(1)} \wedge A_{(1)} - dA_{(2)} \wedge A_{(0)1}, \quad F'_{(4)} = dA_{(3)} - dA_{(2)} \wedge A_{(1)} - F_{(3)1} \wedge \mathcal{A}_{(1)} \end{aligned}$$

while $\mathcal{F}_{(2)} = d\mathcal{A}_{(1)}$ and $F_{(2)1} = dA_{(1)1}$. Upon dualising $F_{(4)}$ to a 1-form field strength $d\chi'$ its kinetic term in the above Lagrangian will be replaced by:

$$-\frac{1}{2}ee^{\frac{3\varphi}{\sqrt{6}}-\frac{\varphi_1}{2}+\frac{3\varphi_2}{2}}(d\chi')^2 + \chi' F'_{(3)} \wedge F'_{(2)} + \chi' F_{(3)1} \wedge \mathcal{F}_{(2)} \tag{B.5}$$

If we perform the field redefinitions:

$$A'_{(1)} = A_{(1)} - A_{(0)1} \wedge \mathcal{A}_{(1)}, \;\; A'_{(2)} = A_{(2)} - A_{(1)1} \wedge \mathcal{A}_{(1)}, \;\; A'_{(2)1} = A_{(2)1} + A_{(1)1} \wedge A'_{(1)}$$

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we find:

$$
\begin{aligned}
F'_{(2)} &= dA'_{(1)} + A_{(0)1} \wedge \mathcal{F}_{(2)}, \qquad F'_{(3)} = dA'_{(2)} - A_{(1)1} \wedge \mathcal{F}_{(2)} \\
F_{(3)1} &= dA'_{(2)1} + dA'_{(1)} \wedge A_{(1)1} - A_{(0)1}(dA'_{(2)} - A_{(1)1} \wedge \mathcal{F}_{(2)}) \\
\chi' F'_{(3)} \wedge F'_{(2)} + \chi' F_{(3)1} \wedge \mathcal{F}_{(2)} &= \chi'(dA'_{(2)} \wedge dA'_{(1)} + dA'_{(2)1} \wedge \mathcal{F}_{(2)})
\end{aligned} \tag{B.6}
$$

Similarly, for the dimensional reduction of Type IIB Lagrangian we obtain:

$$
\begin{aligned}
\mathcal{L}_{5B} = {} & eR - \frac{1}{2}e(\partial\varphi_1)^2 - \frac{1}{2}e(\partial\varphi_2)^2 - \frac{1}{2}e(\partial\varphi)^2 - \frac{1}{2}ee^{2\varphi_1}(\partial\chi_1)^2 - \frac{1}{2}ee^{2\varphi_2}(\partial\chi_2)^2 \\
& - \frac{1}{12}ee^{-\frac{2\varphi}{\sqrt{6}}+\varphi_1-\varphi_2}(F'^{\,RR}_{(3)})^2 - \frac{1}{12}ee^{-\frac{2\varphi}{\sqrt{6}}-\varphi_1-\varphi_2}(F'^{\,NS}_{(3)})^2 - \frac{1}{2}ee^{-\frac{4\varphi}{\sqrt{6}}}\mathcal{F}^2_{(2)} \\
& - \frac{1}{4}ee^{\frac{2\varphi}{\sqrt{6}}+\varphi_1-\varphi_2}(F^{RR}_{(2)1})^2 - \frac{1}{4}ee^{\frac{2\varphi}{\sqrt{6}}-\varphi_1-\varphi_2}(F^{NS}_{(2)1})^2 - \chi_2 dA^{RR}_{(2)} \wedge dA^{NS}_{(1)1} \\
& + \chi_2 dA^{NS}_{(2)} \wedge dA^{RR}_{(1)1}
\end{aligned} \tag{B.7}
$$

where $F^{NS}_{(2)1} = dA^{NS}_{(1)1}$, $\mathcal{F}_{(2)} = d\mathcal{A}_{(1)}$ and:

$$
\begin{aligned}
F'^{\,NS}_{(3)} &= dA^{NS}_{(2)} - dA^{NS}_{(1)1} \wedge \mathcal{A}_{(1)}, \qquad F^{RR}_{(2)1} = dA^{RR}_{(1)1} + \chi_1 dA^{NS}_{(1)1} \\
F'^{\,RR}_{(3)} &= dA^{RR}_{(2)} - dA^{RR}_{(1)1} \wedge \mathcal{A}_{(1)} + \chi_1 dA^{NS}_{(2)} \wedge \mathcal{A}_{(1)}
\end{aligned} \tag{B.8}
$$

As shown in [20], the T-duality rules relating the two truncated theories (B.4) and (B.7) are:

$$
\begin{aligned}
& A_{(0)1} \to \chi_1, \quad A_{(1)1} \to \mathcal{A}_{(1)}, \quad \mathcal{A}_{(1)} \to A^{NS}_{(1)1}, \quad \chi' \to \chi_2, \\
& A'_{(1)} \to A^{RR}_{(1)1}, \quad A'_{(2)} \to A^{NS}_{(2)}, \quad A'_{(2)1} \to -A^{RR}_{(2)}
\end{aligned} \tag{B.9}
$$

together with a rotation of the scalars:

$$
\begin{pmatrix} \varphi_1 \\ \varphi_2 \\ \varphi \end{pmatrix}_{IIA,B} = \begin{pmatrix} \frac{3}{4} & -\frac{1}{4} & -\frac{\sqrt{6}}{4} \\ -\frac{1}{4} & \frac{3}{4} & -\frac{\sqrt{6}}{4} \\ -\frac{\sqrt{6}}{4} & -\frac{\sqrt{6}}{4} & -\frac{1}{2} \end{pmatrix} \begin{pmatrix} \varphi_1 \\ \varphi_2 \\ \varphi \end{pmatrix}_{IIB,A} \tag{B.10}
$$

which takes care of the dilaton couplings of the field strengths.

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